Pomeron-Odderon interference in production of $\pi^+\pi^-$ pairs in ultraperipheral collisions

B. Pire[*], F. Schwennsen[†,*], L. Szymanowski[**] and S. Wallon[†]

[*]CPhT, École Polytechnique, CNRS, 91128 Palaiseau, France
[†]LPT, Université d'Orsay, CNRS, 91404 Orsay, France
[**]Soltan Institute for Nuclear Studies, Warsaw, Poland

Abstract. In this contribution we discuss the production of two pion pairs in high energy photon collisions as they can be produced in ultraperipheral collisions at hadron colliders such as the Tevatron, RHIC or LHC. We find that charge asymmetries may reveal the existence of the perturbative Odderon.

INTRODUCTION

At high energies amplitudes of hadronic reactions with rapidity gaps are dominated by the exchange of a color singlet, *C*-even state – called the Pomeron. In the language of perturbative QCD the Pomeron can be described at lowest order as the exchange of two gluons in the color singlet state. In contrast to the very well settled notion of the Pomeron, the status of its *C*-odd partner – the Odderon – is less safe. Although it is needed *e.g.* to describe properly the different behaviors of pp and $\bar{p}p$ elastic cross sections [1], it still evades confirmation in the perturbative regime, where, again at lowest order, it can be described by the exchange of three gluons in the color singlet state.

The difficulty is rooted in the smaller amplitude for Odderon exchange in comparison to the Pomeron exchange. Hence, in cross sections after squaring the amplitude, the Odderon contribution is always covered by the Pomeron. In this contribution we study charge asymmetries in the production of two pion pairs in photon-photon collisions

$$\gamma(q)\ \gamma(q') \to \pi^+(p_+)\ \pi^-(p_-)\ \pi^+(p'_+)\ \pi^-(p'_-)\,, \tag{1}$$

In such asymmetries, due to interference effects, the Odderon amplitude enters linearly and not quadratically the observable. This approach has been initiated in Ref. [2]. In our specific case we consider the momentum transfer $t = (q - p_+ - p_-)^2$ to provide a hard scale of a few GeV^2 justifying a perturbative calculation within k_T-factorization since at the same time we impose $s \gg |t|$.
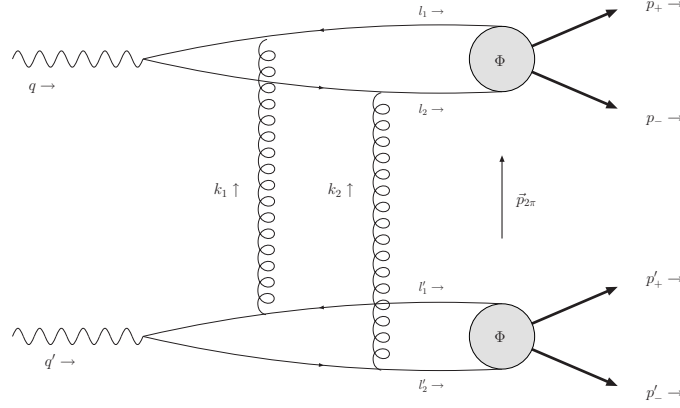
FIGURE 1. Kinematics of the reaction $\gamma\gamma \to \pi^+\pi^- \ \pi^+\pi^-$ in a sample Feynman diagram of the two gluon exchange process.

KINEMATICS, AMPLITUDES AND GDAS

A sample diagram of the two gluon exchange is given in Fig. 1. Due to high energy factorization, the amplitudes can be expressed as convolutions of two impact factors over the transverse momenta of the exchanged gluons. The impact factors themselves consist of a perturbatively calculable part – describing the transition of a photon into a quark-antiquark pair – and a non-perturbative part, the two pion generalized distribution amplitude (GDA) which parametrize the quark-antiquark to hadron transition.

One key point to our final predictions is the choice of the phenomenological input: the GDA [3, 4, 5] which are functions of the longitudinal momentum fraction z of the quark, of the angle θ (in the same rest frame of the pion pair) and of the invariant mass $m_{2\pi}$ of the pion system. After an expansion in Gegenbauer polynomials $C_n^m(2z-1)$ and in Legendre polynomials $P_l(\beta\cos\theta)$ (where $\beta = \sqrt{1 - 4m_\pi^2/m_{2\pi}^2}$) [6], it is believed that only the first terms give a significant contribution:

$$\Phi^{I=1}(z,\theta,m_{2\pi}) = 6z\bar{z}\beta f_1(m_{2\pi})\cos\theta, \tag{2}$$

$$\Phi^{I=0}(z,\theta,m_{2\pi}) = 5z\bar{z}(z-\bar{z})\left[-\frac{3-\beta^2}{2}f_0(m_{2\pi}) + \beta^2 f_2(m_{2\pi})P_2(\cos\theta)\right], \tag{3}$$

where $f_1(m_{2\pi})$ can be identified with the electromagnetic pion form factor $F_\pi(m_{2\pi})$. For the $I=0$ component we use different models. The first model follows Ref. [3] and expresses the functions $f_{0/2}$ in terms of the Breit-Wigner amplitudes of the according resonances. A second model has been elaborated in Ref. [5] and interprets the functions $f_{0/2}$ as corresponding Omnès functions for $S-$ and $D-$waves constructed by dispersion relations from the phase shifts of the elastic pion scattering.

It has been argued [5, 7] that the actual phases of the GDA might be closer to the phases of the corresponding T matrix elements $\frac{\eta_l e^{2i\delta_l} - 1}{2i}$. The third model for the $I=0$ component of the GDA takes this into account by using the technique of model 2 with these phases $\delta_{T,l}$ of the T matrix elements. Indeed, measurements at HERMES [8] do

not observe a resonance effect at the f_0-mass, but concerning the f_2 both phases (δ_2 and $\delta_{T,2}$) are compatible with data [5]. Having this in mind, we consider also a fourth model – a mixed description with the f_0 contribution from model 3 and the f_2 contribution from model 2.

CHARGE ASYMMETRIES

The GDAs for C-even pion pairs ($\Phi^{I=0}$) enter the Odderon exchange amplitude, while those for the C-odd pion pairs ($\Phi^{I=1}$) enter the Pomeron exchange. They are orthogonal to each other in the space of Legendre polynomials in $\cos\theta$ such that only the interference term survives, when the amplitude squared is multiplied by $\cos\theta$ before the angular integration. Thereby we define a charge asymmetry in the following way:

$$
\begin{aligned}
A(t,m_{2\pi}^2,m_{2\pi}'^2) &= \frac{\int \cos\theta \cos\theta' \, d\sigma(t,m_{2\pi}^2,m_{2\pi}'^2,\theta,\theta')}{\int d\sigma(t,m_{2\pi}^2,m_{2\pi}'^2,\theta,\theta')} \\
&= \frac{\int_{-1}^{1} d\cos\theta \int_{-1}^{1} d\cos\theta' \, 2\cos\theta \cos\theta' \, \mathrm{Re}\left[\mathscr{M}_{\mathbb{P}}(\mathscr{M}_{\mathbb{O}}+\mathscr{M}_{\gamma})^*\right]}{\int_{-1}^{1} d\cos\theta \int_{-1}^{1} d\cos\theta' \left[|\mathscr{M}_{\mathbb{P}}|^2 + |\mathscr{M}_{\mathbb{O}}+\mathscr{M}_{\gamma}|^2\right]},
\end{aligned}
\tag{4}
$$

where also the C-odd photon exchange has been included. Since in the kinematic region of interest it is much smaller than the Odderon contribution, the asymmetry is driven by the Odderon/ Pomeron-interference.

The obtained landscape as a function of the two invariant masses would be difficult to measure. To reduce the complexity, we integrate over the invariant mass of one of the two pion systems to obtain

$$
\hat{A}(t,m_{2\pi}^2;m_{\min}^2,m_{\max}^2) = \frac{\int_{m_{\min}^2}^{m_{\max}^2} dm_{2\pi}'^2 \int \cos\theta \cos\theta' \, d\sigma(t,m_{2\pi}^2,m_{2\pi}'^2,\theta,\theta')}{\int_{m_{\min}^2}^{m_{\max}^2} dm_{2\pi}'^2 \int d\sigma(t,m_{2\pi}^2,m_{2\pi}'^2,\theta,\theta')}.
\tag{5}
$$

An analytic calculation of the Odderon matrix element would demand the notion of analytic results for two-loop box diagrams, whose off-shellness for all external legs is different. With the techniques available on the market such a calculation is beyond the scope of this work. Instead we rely on a numerical evaluation by Monte Carlo methods. In particular we make use of a modified version of VEGAS as it is provided by the CUBA library [9].

Although the asymmetry A will problably not be measured, it is illustrative to display it for completeness in Fig. 2. The result for the asymmetry $\hat{A}$ at $t = -1\,\mathrm{GeV}^2$ is shown in Fig. 3. Since our framework is only justified for $m_{2\pi}^2 < -t$, (in fact strictly speaking, one even needs $m_{2\pi}^2 \ll -t$), we keep $m_{2\pi}$ below 1 GeV.

CONCLUSION

We have presented charge asymmetry estimates in production of pion pairs in $\gamma\gamma$ collisions. This asymmetry is linearly dependent on the Odderon amplitude and moreover
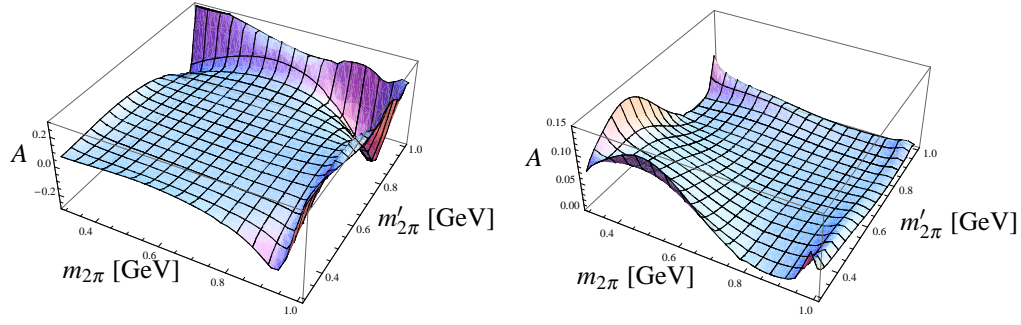
FIGURE 2. Asymmetry A at $t = -1\,\mathrm{GeV}^2$ for model 2 (left) and 3 (right). The shape of model 1 is very similar to model 2, and that of model 4 very similar to model 3.
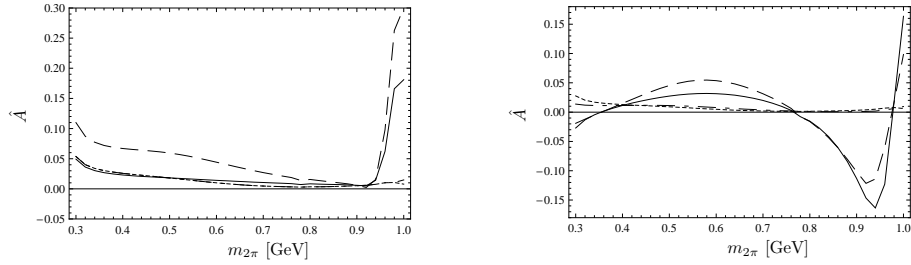

FIGURE 3. Asymmetry $\hat{A}$ at $t = -1\,\mathrm{GeV}^2$ for model 1 (solid), 2 (dashed), 3 (dotted), and 4 (dash-dotted) – model 3 and 4 are nearly on top of each other. Left column has $m_{\min} = .3\,\mathrm{GeV}$ and $m_{\max} = m_\rho$, while right column has $m_{\min} = m_\rho$ and $m_{\max} = 1\,\mathrm{GeV}$.

is sizable but GDA-model dependent. HERMES measurements of two pion electroproduction [8] disfavor models with a strong f_0 coupling to the $\pi^+\pi^-$ state but to our minds higher statistics data, which may come from a JLab experiment at 6 or 12 GeV, are needed before a definite conclusion. As we argue in Ref. [10], in pp collisions at the LHC one can expect of the order of 10^3 events per year. While the rates at RHIC would be far too low, at Tevatron a first search could be possible.

ACKNOWLEDGMENTS

REFERENCES

1. L. Łukaszuk, and B. Nicolescu, *Nuovo Cim. Lett.* **8**, 405–413 (1973); H. G. Dosch, et al., *Eur. Phys. J.* **C24**, 561–571 (2002); A. Breakstone, et al., *Phys. Rev. Lett.* **54**, 2180 (1985).
2. S. J. Brodsky, et al., *Phys. Lett.* **B461**, 114–122 (1999).
3. P. Hägler, et al., *Phys. Lett.* **B535**, 117–126 (2002).

4. P. Hägler, et al., *Nucl. Phys.* **A711**, 232–235 (2002); P. Hägler, et al., *Eur. Phys. J.* **C26**, 261–270 (2002); P. Hägler, et al., *Nucl. Phys. Proc. Suppl.* **121**, 155–159 (2003); M. Diehl, et al., *Phys. Rev. Lett.* **81**, 1782–1785 (1998); M. Diehl, et al., *Phys. Rev.* **D62**, 073014 (2000); D. Y. Ivanov, et al., *PoS* **DIFF2006**, 032 (2006).
5. N. Warkentin, et al., *Eur. Phys. J.* **A32**, 273–291 (2007).
6. M. V. Polyakov, *Nucl. Phys.* **B555**, 231 (1999); N. Kivel, et al., *Phys. Lett.* **B467**, 263–270 (1999).
7. B. Ananthanarayan, et al., *Phys. Lett.* **B602**, 218–225 (2004).
8. A. Airapetian, et al., *Phys. Lett.* **B599**, 212–222 (2004).
9. T. Hahn, *Comput. Phys. Commun.* **168**, 78–95 (2005).
10. B. Pire, et al. (2008), `arXiv:0810.3817[hep-ph]`.